Quark-Antiquark Bound States in an Extended QCD_2 Model*

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Abstract

We study an extended QCD model in $2D$ obtained from QCD in $4D$ by compactifying two spatial dimensions and projecting onto the zero-mode subspace. This system is found to induce a dynamical mass for transverse gluons – adjoint scalars in QCD_2, and to undergo a chiral symmetry breaking with the full quark propagators yielding non-tachyonic, dynamical quark masses, even in the chiral limit. We construct the hadronic color singlet bound-state scattering amplitudes and study quark-antiquark bound states which can be classified in this model by their properties under Lorentz transformations inherited from $4D$.

We study a QCD reduced model in $2D$ which can be formally obtained from QCD in $4D$ by means of a classical dimensional reduction from $4D$ to $2D$ and neglecting heavy K-K (Kaluza-Klein) states. Thus only zero-modes in the harmonic expansion in compactified dimensions are retained. As a consequence, we obtain a two dimensional model with some resemblances of the real theory in higher dimension, that is, in a natural way adding boson matter in the adjoint representation to QCD_2 [3, 4]. The latter fields being scalars in $2D$ reproduce transverse gluon effects [2]. Furthermore this model has a richer spinor structure than just QCD_2 giving a better resolution of scalar and vector states which can be classified by their properties inherited from $4D$ Lorentz transformations. The model is analyzed in the light cone gauge and using large N_c limit. The contributions of the extra dimensions are controlled by the radiatively induced masses of the scalar gluons as they carry a piece of information of transverse degrees of freedom. We consider their masses as large parameters in our approximations yet being much less than the first massive K-K excitation. This model might give more insights into the chiral symmetry breaking regime of QCD_4. Namely, we are going to show that the inclusion of solely lightest K-K boson modes catalyze the generation of quark dynamical mass and allows us to overcome the problem of tachyonic quarks present in QCD_2.

We start with the QCD action in $(3+1)$ dimensions for one flavor (extension to more flavors is straightforward):

$$S_{QCD} = \int d^4x \left[-\frac{1}{2\tilde{g}^2} tr(G^2_{\mu\nu}) + \bar{\Psi}\left(i\gamma^\mu D_\mu - m\right)\Psi \right]. \qquad (1)$$

Follow the scheme of [4] we proceed to make a dimensional reduction of QCD, at the classical level, from $4D$ to $2D$. For this we consider the coordinates $x_{2,3}$ being compactified in a 2-Torus,

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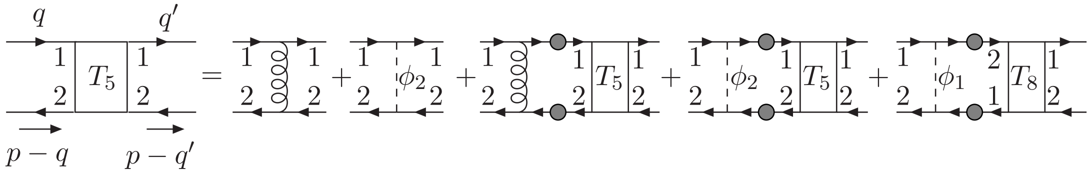

Figure 1: Inhomogeneous Bethe-Salpeter equation for quark-antiquark scattering amplitude T_5.

respectively the fields being periodic on the intervals ($0 \leq x_{2,3} \leq L = 2\pi R$). Next we assume L to be small enough in order to get an effective model in $2D$ dimensions. Then by keeping only the zero K-K modes, we get the following effective action in $2D$, after a suitable rescaling of the fields:

$$
\begin{aligned}
S_2 &= \int d^2x \; tr\Big[-\frac{1}{2}F_{\mu\nu}^2 + (D_\mu\phi_1)^2 + (D_\mu\phi_2)^2\Big] + \bar{\psi}_1\,(i\gamma^\mu\, D_\mu - m)\,\psi_1 \\
&+ \bar{\psi}_2\,(i\gamma^\mu\, D_\mu - m)\,\psi_2 - i\frac{g}{\sqrt{N_c}}\left(\bar{\psi}_1\,\gamma^5\,\phi_1\,\psi_2 + \bar{\psi}_2\,\gamma^5\,\phi_1\,\psi_1\right) \qquad (2) \\
&- i\frac{g}{\sqrt{N_c}}\left(\bar{\psi}_1\,\gamma^5\,\phi_2\,\psi_1 - \bar{\psi}_2\,\gamma^5\,\phi_2\,\psi_2\right) + \frac{g^2}{N_c}\,tr[\phi_1,\phi_2]^2\,,
\end{aligned}
$$

where we have defined the coupling constant of the model $g^2 = N_c\,\tilde{g}^2/L^2$. We expect [5] the infrared mass generation for the two-dimensional scalar gluons ϕ_i. To estimate the masses of scalar gluons ϕ_i we use the Schwinger-Dyson equations as self-consistency conditions, we get:

$$
M^2 = \frac{2N_c\tilde{g}^2}{L^2}\int^\Lambda \frac{d^2p}{(2\pi)^2}\,\frac{1}{p^2+M^2} = \frac{N_c\tilde{g}^2\,\Lambda^2}{8\pi^3}\;\log\frac{\Lambda^2+M^2}{M^2}\,, \qquad (3)
$$

thus M^2 brings an infrared cutoff as expected. We notice that the gluon mass remains finite in the large-N_c limit if the QCD coupling constant decreases as $1/N_c$ in line with the perturbative law of $4D$ QCD. We adopt the approximation $M \ll \Lambda \simeq 1/R$ to protect the low-energy sector of the model and consider the momenta $|p_{0,1}| \sim M$. Thereby we retain only leading terms in the expansion in p^2/Λ^2 and M^2/Λ^2, and also neglect the effects of the heavy K-K modes in the low-energy Wilson action. We observe that the limit $M \ll \Lambda$ supports consistently both the fast decoupling of the heavy K-K modes and moderate decoupling of scalar gluons [4], the latter giving an effective four-fermion interaction different from [6]. Also allow us to define the "heavy-scalar" expansion parameter $A = g^2/(2\pi\,M^2) = 1/log\frac{\Lambda^2}{M^2} \ll 1$. Now we proceed to the study of bound states of quark-antiquark. In our reduction we have four possible combinations of quark bilinears to describe these states with valence quarks: $(\psi_1\,\bar{\psi}_1), (\psi_1\,\bar{\psi}_2)$, $(\psi_2\,\bar{\psi}_1), (\psi_2\,\bar{\psi}_2)$. We need to compute the full quark-antiquark scattering amplitude T in the different channels. As an example we are going to show the computing of T_5 which correspond to the scattering $(q_1 + \bar{q}_2 \longrightarrow q_1 + \bar{q}_2)$. It satisfies, the equation given graphically in Figure 1, in the large N_c limit and in ladder exchange approximation (non-ladder contribution are estimated to be of higher order in the A expansion). Notice that in the equation for T_5 the amplitude T_8 appears, which correspond to the process $q_2 + \bar{q}_1 \longrightarrow q_1 + \bar{q}_2$. This means that the equations for T_5 and T_8 are coupled. In Figure 1 all internal fermion lines correspond to dressed quark propagators which were determined in [4] by using the large N_c limit and the one-boson exchange

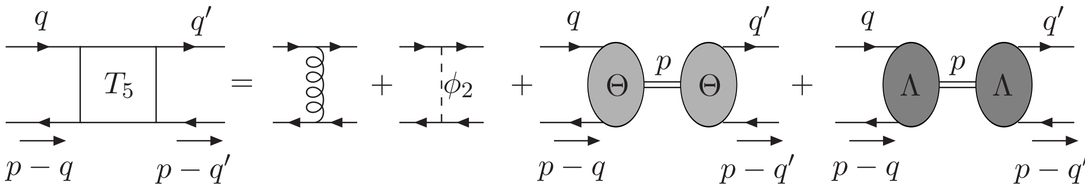

Figure 2: $q\bar{q}$ scattering T_5 in the color-singlet channel.

approximation. There are two kind of solutions for the dressed quark propagators. In one solution, the perturbative one, we have tachyonic quarks in the chiral limit as in QCD_2 and our model could be interpreted as a perturbation from the result of chiral QCD in $2D$. But we are not allowed to consider that possibility because the spectrum for the lowest $q\bar{q}$ bound states becomes imaginary if one takes into account the scalar field exchange. The second solution, the non-perturbative one, supports non-tachyonic quarks with masses going to zero, in the chiral limit and also yield real masses for the $q\bar{q}$ bound states. Having found the full quark propagator we proceed to solve the inhomogeneous Bethe-Salpeter equation for T_5 and T_8. We obtain for T_5 (see Figure 2):

$$\begin{aligned} \overset{\alpha\beta,\gamma\delta}{T_5}(q,q';p) &= -i\frac{g^2}{N}\,\frac{\gamma_-^{\alpha\delta}\,\gamma_-^{\beta\gamma}}{(q-q')_-^2} - i\frac{g^2}{N}\,\frac{\sigma_3^{\alpha\gamma}\,\sigma_3^{\beta\delta}}{[(q-q')^2 - M^2 + i\epsilon]} + \\ &+ \sum_j \frac{i}{(p^2-\bar{m}_j^2+i\epsilon)}\,\Theta_j^{\alpha\gamma}(q;p)\,\Theta_j^{\beta\delta}(q';p) + \sum_k \frac{i}{(p^2-\tilde{m}_k^2+i\epsilon)}\,\Lambda_k^{\alpha\gamma}(q;p)\,\Lambda_k^{\beta\delta}(q';p)\,. \end{aligned} \tag{4}$$

There are no continuum states in the quark-antiquark amplitude– only bound states at $p^2=\bar{m}^2$ and $p^2=\tilde{m}^2$, whose residue yield the bound state wave functions $\Theta(p,r)$ and $\Lambda(p,r)$. These bound states have a direct interpretation in term of Dirac Bilinears of the theory in $4D$ [4]. In particular, the pseudoscalar $4D$ states are related with $\Lambda_k(r;p)$ which could be interpreted as the vertex: (quark)-(antiquark)-($4D$ pseudoscalar meson). The masses of the bound states and the vertex functions are fixed by the solution of the homogeneous Bethe-Salpeter equation which, in our model, yield to a eigenvalue problem, generalization of the integral equation found by 't Hooft [1] in QCD_2. For example the following eigenvalue integral equation determine the mass spectrum of the pseudoscalar bound states (in the chiral limit):

$$\begin{aligned} \tilde{m}^2\,\phi(x) = &-\frac{g^2}{\pi}\int_0^1 dy\,\frac{\mathbf{P}}{(x-y)^2}\,\phi(y) - A\,\frac{\Sigma_0^2}{x^{1-\beta}}\int_0^1 dy\,\frac{\phi(y)}{(1-y)^{1-\beta}} \\ &-A\,\frac{\Sigma_0^2}{(1-x)^{1-\beta}}\int_0^1 dy\,\frac{\phi(y)}{y^{1-\beta}} + A\,\frac{\Sigma_0^2}{[(1-x)\,x]^{1-\beta}}\int_0^1 dy\,\phi(y) + A\,\Sigma_0^2\int_0^1 dy\,\frac{\phi(y)}{[(1-y)\,y]^{1-\beta}}, \end{aligned} \tag{5}$$

where $\beta=A/2$. To explore solutions to Eq.(5) we examine small A. Evidently Eq.(5) does not mix even and odd functions with respect to the symmetry $x \Longleftrightarrow 1-x$. On the other hand the ground state should be an even function. When inspecting the wave function end-point asymptotics from the integral equation (5) one derives the following even function as a ground state solution for $A\to 0$ limit: $\phi_0(x) = (4x[1-x])^{\frac{A}{2}} - \frac{1}{\pi}(4x[1-x])^{\frac{1}{2}}$. This is basically a non-perturbative result that differs from 't Hooft solution in the $A\to 0$ limit, giving $p^2=0$,

Pseudoscalar	Scalar
0	974
1286	1531
1741	1929
2166	2356

Table [1]: Some values for the masses of bound states in $[MeV]$, where we have taken $\frac{g}{\sqrt{\pi}} = 267[MeV]$ and $A = 0.22$.

as we would expect from spontaneous chiral symmetry breaking in $4D$. For the other massive states we are unable to find analytic solutions, as happens with 't Hooft equation, but we could estimate them working with the Hamiltonian matrix elements $\tilde{m}^2(\phi,\phi) = (\phi, H\phi)$ and using the regular perturbation theory, starting from 't Hooft solutions ($A = 0$). In Table [1] we show lowest values of mass spectra for scalar and pseudoscalar states.

Discussions

Quantum Chromodynamics at low energies has been decomposed by means of dimensional reduction from $4D$ to $2D$ and a low energy effective model in $2D$ has been derived. In this model we did an explicit analysis of meson bound states by solving the inhomogeneous and the homogeneous Bethe-Salpeter equations, in the large N_c limit and in the ladder exchange approximation. We found that the $2D$ model has four types of bound states which can be classified by their properties under Lorentz transformations inherited from $4D$. The $4D$ pseudoscalar and scalar sectors of the theory were analyzed and in the quiral limit a massless solution for the pseudoscalar ground state was found. We interpreted this solution as the "pion" of the model. Also, in solving the Bethe-Salpeter equations we found a solution to the full quark propagators yielding non-tachyonic dynamical quark masses, in contrast to what happen in QCD_2.

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